Math 113-012, Exam 3
28-30 October 2010
D. G. Wright

Name ______________________
Row ______________________

Show work. Each problem or part of problem is worth 5 points.

1. Find the surface area when the line segment from $(4, 0)$ to $(16, 5)$ is rotated about the y- axis.

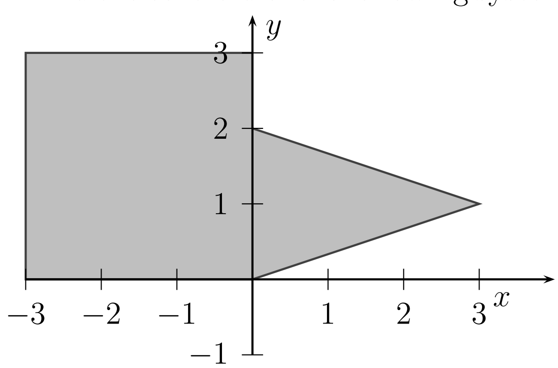
   Use the formula for the surface area of a lampshade $SA = 2\pi \frac{R_1 + R_2}{2} \ell$ where R_1 and R_2 are the two radii and ℓ is the slant height. In this case $R_1 = 4$, $R_2 = 16$ and $\ell = 13$. The answer is 260π.

2. The curve $y = \sqrt{4 - x^2}$, $-1 \le x \le 1$, is rotated about the x-axis. Find the area of the resulting surface.

   $\frac{dy}{dx} = \frac{-x}{\sqrt{4-x^2}}$. The surface area is $2\pi \int_{-1}^{1} \sqrt{4-x^2}\sqrt{1 + \frac{x^2}{4-x^2}}dx$

   $= 2\pi \int_{-1}^{1} \sqrt{4-x^2}\sqrt{\frac{4}{4-x^2}}dx = 4\pi \int_{-1}^{1} dx = 8\pi$

3. Find the centroid of the following system consisting of a square and an isosceles triangle.



   $A = 9 + 3 = 12$

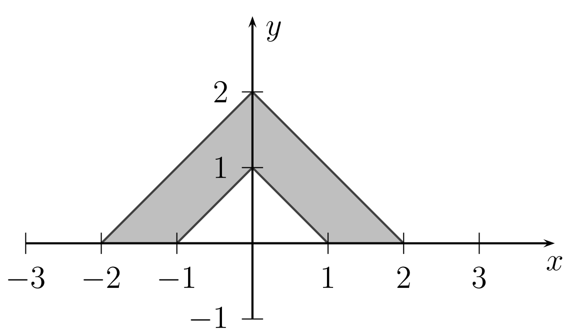
   $M_y = -\frac{3}{2} \cdot 9 + 1 \cdot 3 = -\frac{21}{2}$

   $M_x = \frac{3}{2} \cdot 9 + 1 \cdot 3 = \frac{33}{2}$

   $\overline{x} = \frac{M_y}{A} = -\frac{7}{8}$

   $\overline{y} = \frac{M_x}{A} = \frac{11}{8}$

4. Find the centroid of the region between the two triangles in the x-y plane. You may use either Hint 1 or Hint 2. Hint 1: The area can be found as the difference of two areas. In a similar manner, the moment about the x-axis can be found as the difference of two moments. Hint 2: Use the Theorem of Pappus.



   $A = 4 - 1 = 3$

   $M_x = \frac{2}{3}4 - \frac{1}{3}1 = \frac{7}{3}$

   $\overline{y} = \frac{M_x}{A} = \frac{7}{9}$

   $\overline{x} = 0$ by symmetry

5. Evaluate the following limits if they exist. If the limit does not exist, so state.

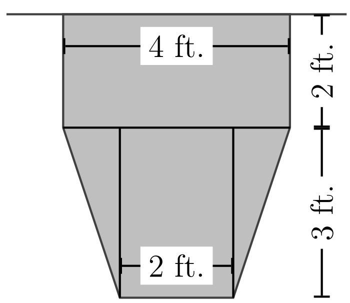
   (a) $\lim_{n\to\infty} \frac{1}{n} = \underline{\qquad 0 \qquad}$

   (b) $\lim_{n\to\infty} \left(1+\frac{5}{n}\right)^n = \underline{\qquad e^5 \qquad}$

   (c) $\lim_{n\to\infty} \frac{\sqrt{n^5+2n^3+5}}{n^3} = \underline{\qquad 0 \qquad}$

6. Define $\sum_{n=1}^{\infty} a_n = L$. Let $S_n = a_1 + a_2 + a_3 + \cdots a_n$. Then $\sum_{n=1}^{\infty} a_n = L$ means that $\lim_{n\to\infty} S_n = L$.

7. What is the hydrostatic force on the given plate whose top is at the surface of the water if the density of water is δ lbs/ft^3?



Depth of centroid of 2×4 rectangle = 1ft.

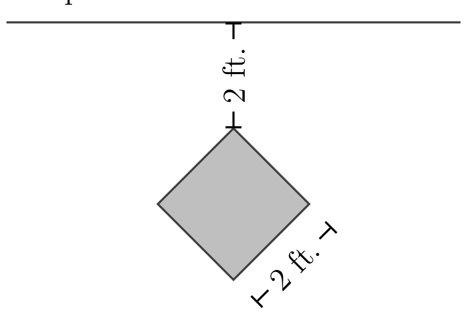
Depth of centroid of 2×3 rectangle = $\frac{7}{2}$ ft.

Depth of centroid of triangles = 3 ft.

Force = $[1 \cdot 8 + \frac{7}{2} \cdot 6 + 3 \cdot (\frac{3}{2} + \frac{3}{2})]\delta$ lbs

= 38δ lbs

8. What is the hydrostatic force on a 2 foot by 2 foot square diamond aquarium window whose top is 2 feet below the surface of the water if the density of water is δ lbs/ft^3?



Diagonal of square = $2\sqrt{2}$ ft. Centroid of square is $2+\sqrt{2}$ ft below the surface. Area of square = 4 ft^2. Force = $(2+\sqrt{2}) \cdot 4\delta$ lbs = $(8+4\sqrt{2})\delta$ lbs

9. If $0 < r < 1$, prove that $\lim_{n\to\infty} r^n = 0$. Multiply each side of the inequality by r^n to get $0 < r^{n+1} < r^n$. So the sequence r^n is decreasing and bounded below by 0. So by a theorem, it must converge to some number L. But $\lim_{n\to\infty} r^{n+1} = L$ and $\lim_{n\to\infty} r^{n+1} = rL$. So $rL = L$. Since $r \neq 1$, we must have $L = 0$.

10. Find the fifteenth partial sum S_{15} for the series $\sum_{n=1}^{\infty}(-1)^{n+1}$.

An even partial sum has the same number of $1's$ and $-1's$ so the partial sum is 0. An odd partial sum has one more $= 1$ than -1 so the partial sum is 1.

11. Determine whether each series converges or diverges. If it converges, give its sum.

(a) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}} =$ ____________ Diverges because the terms go to $1 \neq 0$.

(b) $\sum_{n=1}^{\infty} \frac{2}{4n^2-1} =$ ____________ Use partial sum decomposition to get $\frac{2}{4n^2-1} =$ Which is seen to be $\frac{1}{2n-1} - \frac{1}{2n+1}$. The series is now seen to be telescoping $1 - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \cdots$ its sum is 1.

(c) $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n} =$ ____________

This is a geometric series with first term $\frac{4}{3}$ and ratio $\frac{2}{3}$. The sum is $\frac{\frac{4}{3}}{1-\frac{2}{3}} = 4$

12. Determine whether each series converges or diverges. State any convergence/divergence tests you use. For the Integral Test, evaluate the appropriate integral. For the Comparison Test or Limit Comparison Test give the appropriate comparison series.

(a) $\sum_{n=1}^{\infty} ne^{-n^2}$ Converges by Integral Test or Ratio Test. Check details.

(b) $\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$ Converges by Integral Test or by Comparison Test with the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ since $\ln n < n$ for large n. Check details.

(c) $\sum_{n=1}^{\infty} \frac{n^2+3n+1}{n^3+2n^2+n+1}$ Diverges by Limit Comparison Test with the harmonic series. Check details.

(d) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+2n^2+n+1}}$ Converges by either the Limit Comparison Test or the Comparison Test with the series $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$. Check details.

(e) $\sum_{n=1}^{\infty} \frac{\sin(\frac{1}{n})}{\sqrt{n}}$ Converges by Limit Comparison test with the series $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$. Check details.